Preliminaries for topological change detection using sensor networks

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Abstract. Topological changes to regions, such as merging/splitting and hole formation/elimination, are significant events in their evolution. Information about such salient changes is useful in many applications. The research reported in this paper provides theoretical foundations for such topological change detection in sensor networks. A local tree model is proposed in the spatial domain, based on which a set of types of topological changes is specified. We also present a sensor network framework which captures the necessary information required by the tree model. Both the local tree model and the sensor network framework form the foundations for detection approaches that allow sensor networks to report topological changes.

1 Introduction and background

Wireless sensor network technology provides real-time information about the environment, and this will play an important role in the monitoring of geographic phenomena. Most sensing applications up to now have focused on capturing, processing and reporting geographical information in the form of spatial-temporal data. However, topological changes to regions, such as merging/splitting and hole formation/elimination, are often the significant events, and in many applications it is useful to have information about such topological changes. For example, in the case of wildfire, fire fighters might be interested if the fire zone regions *split* and become disconnected, so that they can reorganize the team accordingly. They might also be interested in *merging* fires, as it sometimes slows down the burn when the fires are burning over each other. This paper focuses on addressing the basic issues related to the application of sensor networks to the detection of topological changes.

Topological features are important aspects of spatial data. Their representations are the focus of much research work in spatial data modeling. Topological features allow us to classify spatial changes. Egenhofer and Al-Taha [1] analyze and classify the spatial changes involving two regions based on their topological relations before and after the change, and the result is recorded using the conceptual neighborhood graph. In previous work [2], we have proposed a model that represents the dynamic topology of an areal object (a collection of region components, possibly with holes and islands), based on which different types of topological changes are specified.

A straightforward application of sensor networks is to the monitoring of geographical phenomena [3]. Previous research either focuses on proposing energy-efficient approaches to transmitting entire sensing data back to base stations [4, 5], or focuses on providing important spatial properties of the phenomena. For example, the snake model proposed by Jin and Nittel [6] is able to derive the area and centroid of a deformable 2D object over time. Recently, there is an increasing interest in considering topological information when processing sensing data. Gandhi, Hershberger and Suri [7] emphasize the topology of the isolines in a scalar field and propose an approach that approximate a family of isolines by a collection of topology-preserving polygons. Sarkar *et al.* [8] present a distributed algorithm for the construction of a contour tree to represent the topological structure of contours in a scalar field, based on which isoline queries can be enabled. Worboys and Duckham [9] provide a computational model for sensor networks to detect global high-level topological changes based on low-level 'snapshot' of spatiotemporal data.

Current research has made initial attempts to topological change detection using wireless sensor networks. Farah and Cheng *et al.* [10] provided initial attempts to detect topological changes in responsive sensor networks by an event-driven approach. Sadeq [11] proposed the idea of detecting topological changes by maintaining the boundary state of areal objects. In [12], we presented the topological change detection approach based on local aggregation.

2 Local tree model

This work is concerned with evolving areal objects in the plane. An areal object is a collection of region components, possibly with holes or islands. At a particular time, an areal object can be considered as a set R of points in the spatial domain, and the evolution of an areal object can be considered as a temporal sequence of snapshots of the areal object. Each pair of consecutive snapshots describes a change, called a *basic transition*. Topological changes can be observed during a basic transition.

Let R_1 and R_2 be a pair of areal objects that define a basic transition, where R_1 and R_2 are the start and end snapshots, respectively. Any location p in the spatial domain must have one of the following four states: (I) $p \notin R_1$ and $p \in R_2$. (II) $p \in R_1$ and $p \notin R_2$. (III) $p \in R_1$ and $p \in R_2$, or (IV) $p \notin R_1$ and $p \notin R_2$. Based on the states of each location, the whole spatial domain can be partitioned into several components. Each component is a maximal topologically connected set of locations in the same state.

The locations in state I or state II form the components that are either added to or removed from the areal object during the basic transition. We call such components *transition regions*. For simplicity, we assume each basic transition has only one transition region that is topologically equivalent to a disk; i.e., the transition region is a single piece without any holes. The locations in state III or IV form the components that do not change during the basic transition. The structure of such components is important to determine the type of the basic transition. However, not all of the components are crucial in the determination of the type of topological change. Only the components that are adjacent to the transition region are necessary. These components are referred to as *C-components.*

As an example, Fig. 1(a) shows a basic transition of an areal object, based on which the spatial domain is partitioned into five components *A*-*E*, as shown in Fig. 1(b). Among these components, component *C* is the transition component, and components *A*, *B* and *D* are C-components.

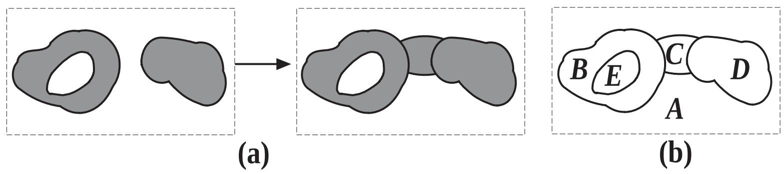

Fig. 1. A basic transition

To specify topological changes, we are most interested in the adjacency relations and the surrounded-by relations between the C-components. These are defined as follows:

Definition 1. *Let X_1 and X_2 be a pair of components in a basic transition.*

1. *X_1 is said to be* adjacent to *X_2 if the boundary of X_1 intersects the boundary of X_2.*
2. *X_1 is said to be* surrounded by *X_2 if any path that connects a point in the closure of X_1 to a point at infinity intersects the closure of $X_2 \backslash X_1$. X_1 is said to* surround *X_2, if X_2 is surrounded by X_1.*

The structure of the C-components in a basic transition have the properties stated as follows, and section 4 provides detailed proofs of these properties.

1. There is exactly one C-component X which surrounds all the other C-components. X is referred to as the *background C-component* of the basic transition.
2. The topological structure of the C-components in a basic transition can be represented by a rooted tree. A vertex of the tree represents a C-component, and an edge of the tree connects a pair of vertices representing adjacent C-components. The root of the tree represents the background C-component.

As different rooted trees can be explored in a systematic way, we are able to generate the possible topological structures between the C-components of a basic transition. Fig. 2 lists all the rooted trees with less than 4 vertices, and examples of structures represented by the rooted trees are also provided. In the figure, the transition region is indicated by shaded area, and the vertices of the representation tree are placed inside the C-components they represent.

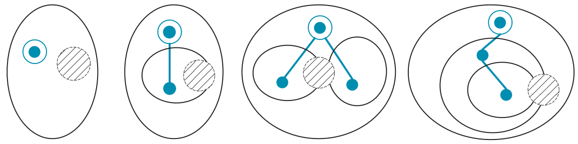

Fig. 2. Tree representations for different configurations of C-components

Local topological structure	State of transition region (T) / State of background C-component (B)			
	T = I B = IV	T = II B = IV	T = I B = III	T = II B = III
	Region appear (A)	Region disappear (B)	Hole disappear (I)	Hole appear (J)
	Topology-preserving changes (C, D, K, L)			
	Regions merge (E)	Region split (F)	Hole split (M)	Hole merge (N)
	Region self-merge (G)	Region self-split (H)	Hole self-split (O)	Hole self-merge (P)

Fig. 3. Classification of basic transitions

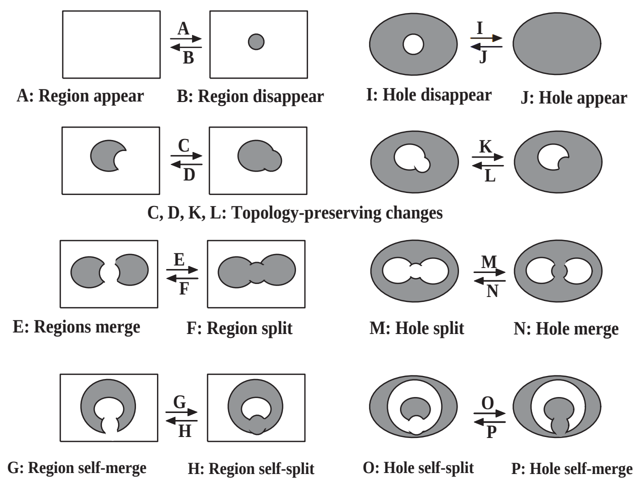

Fig. 4. Examples of specific types of topological changes

The classification of a basic transition is based on the following three factors: (1) The topological structure of its C-components, (2) the state of the locations in its transition region, and (3) the state of the locations in its background C-component. The classification yields different types of topological changes incurred by a basic transition. Fig. 3 shows the classification results, in which the 13 types of specific topological changes are distinguished, and Fig. 4 provides an example of a basic transition for each type of topological change. These types of topological changes will be used in the sensor report to describe the observed basic transition.

3 Completeness of local tree model

In this section, we prove that the structure of C-components in any basic transition can be represented by a rooted tree, and therefore the local tree model provides a complete coverage over all basic transitions.

The following lemmas and theorems use the notion of 'partially surrounded-by'. Let X_1 and X_2 be a pair of distinctive C-components in a basic transition, and let T be the transition region. X_1 is defined to be *partially surrounded by* X_2 (or X_2 *partially surrounds* X_1) if X_1 is surrounded by $T \cup X_2$. Fig. 5 shows an example in which C-component X_1 is partially surrounded by C-component X_2.

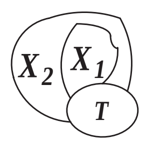

Fig. 5. Example of partially-surrounded-by relation

We first present several lemmas to show the properties of the C-components in a basic transition. As the proofs of these lemmas are long, we do not include them in this paper. These proofs can be found in [13].

Lemma 1 *The adjacency graph of the C-components in a basic transition is connected.*

Lemma 2 *Let X_1 and X_2 be a pair of adjacent C-components in a basic transition. One of the following statements must be true: either X_1 partially surrounds X_2, or X_2 partially surrounds X_1.*

Lemma 3 *Let X_1, X_2, and Y be distinct components in the spatial domain such that $X_1 \cup Y$ surrounds X_2 and $X_2 \cup Y$ surrounds X_1. Y must surround both X_1 and X_2.*

Lemma 4 *Let X_1 and X_2 be a pair of C-components. The following statements cannot both be true: (1) X_1 is partially surrounded by X_2. (2) X_2 is partially surrounded by X_1.*

Lemma 5 *Let X_1, X_2 and Y be distinct C-components. The following statements cannot both be true: (1) X_1 is adjacent to Y and partially surrounds Y. (2) X_2 is adjacent to Y and partially surrounds Y.*

Theorems 1 and 2 present the major results of this section.

Theorem 1. *The adjacency graph of the C-components in a basic transition is a tree.*

Proof. This can be proved by showing that the adjacency graph of the C-components in a basic transition is both connected and cycle-free.

By Lemma 1, the adjacency graph is connected. We prove it is cycle-free by contradiction. Suppose we are able to find a cycle in the adjacency graph, which must contain more than 2 vertices. By Lemma 2, the partially-surrounded-by relation must hold between the C-components represented by any pair of adjacent vertices in the cycle, and there are two possible cases to consider:

Case 1: there are three consecutive vertices in the cycle representing C-components X_{i-1}, X_i, and X_{i+1}, respectively, such that both X_{i-1} and X_{i+1} are adjacent to X_i and partially surround X_i. This contradicts the conclusion of Lemma 5.

Case 2: for $\forall i$, $i = 1, 2, ..., k - 1$ (where k is the number of vertices in the cycle), X_i partially surrounds X_{i+1}, and X_k partially surrounds X_1. It follows that X_1 partially surrounds itself. Hence by definition, the transition region surrounds X_1. This contradicts the assumption that the transition region is simply connected.

Both cases lead to contradictions, so the adjacency graph of the C-components is both connected and cycle-free, and therefore must be a tree.

□

Theorem 2. *Among all the C-components of a basic transition, there must be exactly one C-component that surrounds all the other C-components.*

Proof. It is straightforward to prove this theorem, if there is one or two C-components in the basic transition. Consider the basic transitions which have more than two C-components.

First, we show that there must be at least one C-component B that partially surrounds all the other C-components. If not, we are able to find two C-components X_n and X_m such that neither of them is partially surrounded by any other C-components. By theorem 1, the adjacency graph of these C-components is a tree. By Lemma 2, in the path of the adjacency tree that connects the vertices representing X_n and X_m, there must be three consecutive vertices that represent C-components X_{i-1}, X_i and X_{i+1}, such that both X_{i-1} and X_{i+1} are adjacent to X_i and partially surround X_i. This contradicts the result of Lemma 5. Therefore, there must be at least one C-component B that partially surrounds all the other C-components.

In addition, by Lemma 4, there is at most one C-component B that partially surrounds all the other C-components. In all, there is exactly one C-component B that partially surrounds all the other C-components.

Finally, we show that B surrounds all the other C-components. Let T be the transition region, and U be the union of all the C-components except for

B. As B partially surrounds all the other C-components, it follows that $B \cup T$ surrounds U. In addition, by the definition of C-components, $B \cup U$ surrounds T. By Lemma 3, we have B surrounds U, and therefore B surrounds all the other C-components.

□

Theorems 1 and 2 show that the structure of C-components in any basic transition can be represented by a rooted tree, in which the root represents the unique background C-component.

4 Sensor network configuration

We are using sensor networks to track and report topological changes. This section provides the basic assumptions we have made about the sensor networks, as well as the definitions of basic elements in sensor networks that are approximate representations of the components and relations required by the local tree model.

We assume that a node located near the boundary of the sensing area is selected to be the *reference node*, which is assumed to be located outside the scope of the observing phenomena. The sensor nodes in the sensing area induce a Voronoi diagram, and each sensor node n is associated with a Voronoi cell consisting of all the locations that are closer to n than to any other sensor node.

The sensor nodes take measurements at a sequence of sampling rounds t_0, t_1, ..., t_n. We assume that the reading of a sensor node at any of the sampling rounds is either 0 or 1. Our interpretation is that the reading is 1 if the sensor node is in an area of high intensity (reading above a threshold), otherwise it is 0. A change is captured by sensor readings at a pair of consecutive sampling rounds, and the type of a transition is determined by comparing the readings. The comparison first defines four states of nodes at sampling round t_i.

Definition 2. *Let* $r(n,t) \in \{0,1\}$ *denote the reading of a node* n *at a time* t. *The* state *of* n *at a sampling round* $t_i (1 \leq i \leq k)$ *is defined to be a pair* $h = (r(n,t_{i-1}), r(n,t_i))$, *such that* $h \in \{(0,1),(0,0),(1,0),(1,1)\}$.

The states of the sensor nodes together with the sensor connectivity yield the following concepts that approximate the fundamental properties required by the local tree model.

Definition 3. *Let* N *be a set of sensor nodes,* N *is said to be a* homogeneous sensor component *if the nodes in* N *are in the same state and induce a connected component in the communication graph. Moreover,* N *is defined to be a* maximal homogeneous sensor component, *if it is impossible to find a node* n *in the sensing area such that (1)* $n \notin N$, *and (2)* $N \cup \{n\}$ *is a homogeneous sensor component.*

Definition 4. *Let* N_1 *and* N_2 $(N_1 \cap N_2 = \emptyset)$ *be a pair of homogeneous sensor components.*

1. *N_1 is said to be* adjacent to *N_2 if there are nodes $n_1 \in N_1$ and $n_2 \in N_2$ such that n_1 and n_2 are direct neighbors in the communication graph. Otherwise, N_1 and N_2 are said to be* separated.
2. *N_1 is said to be* surrounded by *N_2, if any path in the communication graph that starts from the reference node and contains a node of N_1 must contain a node of N_2. N_1 is said to* surround *N_2, if N_2 is surrounded by N_1.*

Definition 5. *Let N be a maximal homogeneous sensor component.*

1. *N is defined to be a* transition sensor component*, if N consists of only nodes either in state $(0,1)$ or in state $(1,0)$.*
2. *N is defined to be a* sensor C-component*, if both of the following conditions are satisfied: (1) N consists of nodes either in state $(0,0)$ or in state $(1,1)$, and (2) N is adjacent to transition sensor component.*
3. *N is defined to be a* background sensor C-component*, if it is a sensor C-component and it surrounds all the other sensor C-components.*

Table 1. Elements in spatial domain and their approximations in sensor networks

In spatial domain	**In sensor networks**
A C-component in state III	A sensor C-component in state $(1,1)$
A C-component in state IV	A sensor C-component in state $(0,0)$
The transition region	The transition sensor component
The background C-component	The background sensor C-component
Adjacency relations between C-components	Adjacency relations between sensor C-components
Surrounded-by relations between C-components	Surrounded-by relations between sensor C-components

Table 1 shows the correspondences between the components and relations we defined in spatial domain and their representations in sensor networks. Based on the correspondences, all the concepts defined in the spatial domain can be represented and computed in terms of states of sensors and connectivity between them.

Ideally, the elements defined in sensor networks represent the properties of the areal objects in the spatial domain. The nodes located in a component of the spatial domain form exactly one maximal homogeneous sensor component. Components in the spatial domain are adjacent if and only if they are represented by adjacent maximal homogeneous sensor components. Components in the spatial domain surround each other if and only if they are represented by maximal sensor components that surround each other. However, such a perfect matching may not always exist. Inconsistency may be caused by low node density and improper setting of communication ranges. Here are some of the examples.

First, if the density of the nodes is low, a component in the spatial domain that is small enough may not contain any sensor node, and therefore is not represented by any sensor component, as shown in Fig. 6(a).

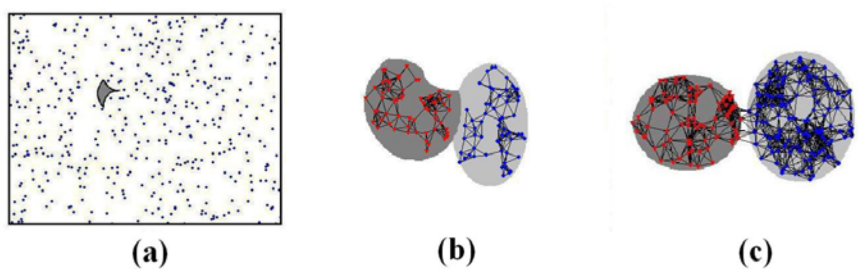

Fig. 6. Configurations that lead to errors in reports

Second, if the communication range of the sensor nodes is not large enough, inconsistencies may occur. For example, a pair of adjacent components in the spatial domain are represented by a pair of separated maximal homogeneous sensor components, as shown in Fig. 6(b).

Finally, if the communication range of the sensor nodes is not small enough, inconsistencies may occur. For example, a pair of components that are not adjacent in the spatial domain is represented by a pair of adjacent maximal homogeneous sensor components, as shown in Fig. 6(c).

In order to avoid inconsistency, we stipulate that the sensor node deployment satisfies the following constraints: **(1) Density constraint,** sensor nodes are deployed densely enough so that a sensor node measurement accurately reflects all locations in its Voronoi cell. **(2) Communication constraint,** each sensor node communicates exactly with the nodes in its adjacent Voronoi cells.

Sensor networks that conform to both constraints are able to provide correct sensing reports on the topological changes based on the framework we provided. Failure to conform to both constraints does not disable the whole detection approach, but it might result in errors in the reports of some sensing rounds. For example, sensor network might report that splitting of a wild fire is observed in a sensing round, but in reality the fire does not split.

5 Conclusion and future work

This paper provides the computational foundations for topological change detection in sensor networks. Based on the local tree model, basic transitions are classified into a set of classes, and each class specifies a type of topological change. We also analyze the corresponding elements required by the local tree model in the sensor network configuration. Based on these foundations, we have developed distributed algorithms for topological change detection using sensor networks, detailed in [13].

Future work includes the following areas: (1) extensions of current research in order to handle non-incremental transitions, in which more than one transition region exists in a pair of consecutive snapshots, and each transition region may have holes. (2) design of algorithms for topological change detection under imperfect information, which can be introduced in either by uncertainty in sensor readings or by improper configuration of sensor networks.

Acknowledgments

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